

For each of these four relations, decide whether on any domain of the relevant kind, it is always:

- 1) Reflexive $\forall x R(x,x)$ or Irreflexive $\forall x \neg R(x,x)$ or neither
- 2) Symmetric $\forall x \forall y (R(x,y) \rightarrow R(y,x))$ or Asymmetric $\forall x \forall y (R(x,y) \rightarrow \neg R(y,x))$ or Antisymmetric $\forall x \forall y [(R(x,y) \wedge R(y,x)) \rightarrow x=y]$ or none
- 3) Transitive $\forall x \forall y \forall z [(R(x,y) \wedge R(y,x)) \rightarrow R(x,z)]$ or Anti-transitive $\forall x \forall y \forall z [(R(x,y) \wedge R(y,x)) \rightarrow \neg R(x,z)]$ or neither

Relation 1)

Let Domain = some set of people in the world

Let Loves(x,y) mean that x loves y

Neither reflexive nor irreflexive, not symmetric, asymmetric, or anti-symmetric, and neither transitive nor anti-transitive

Relation 2)

Let Domain = some set of people in the world

Let Taller(x,y) mean that x is taller than y

irreflexive, asymmetric and anti-symmetric, and transitive

Relation 3)

Let Domain = some set of natural numbers

Let LessEqual mean that x is less than or equal to y

Reflexive, anti-symmetric, and transitive

Relation 4)

Let Domain = some set of all people in the world

Let Ancestor(x,y) mean that x is a direct ancestor of y

irreflexive, asymmetric and anti-symmetric, and transitive

---There any intuitive axioms that are missing from some of these

1) Find a sentence that is true that contains no names/constants where the only predicate is LessEqual

$\forall x \forall y (\text{LessEqual}(x,y) \vee \text{LessEqual}(y,x))$

Adding this to anti-symmetry + transitivity (reflexivity follows) gives the axioms for a total order.

2) Find a sentence that is true that contains no names/constants where the only predicate is Taller

If we assume that two people really can be the same height (say everyone's height is rounded to the nearest inch) then there are more things that are true about Taller. For example, $\forall x \forall y \forall z [(\neg \text{Taller}(x,y) \wedge \neg \text{Taller}(y,z)) \rightarrow \neg \text{Taller}(x,z)]$ This means that 'not taller than' is transitive. If $\neg \text{Taller}(x,y)$ implied $\text{Taller}(y,x)$, then (with asymmetry) this would be the same as transitivity. Adding this gives the axioms for a strict weak ordering.